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B.Sc (part-1) Hons

Paper-1

Fourier series and Fourier coefficients.

Fourier series:- Any finite, continuous single-valued periodic function can be expressed as a summation of simple harmonic terms having frequencies which are multiples of that of the given function. A periodic function $f(x)$ defined in the interval $(-\pi, \pi)$ i.e for $-\pi < x < \pi$ and having a period 2π can be expressed as:

$$f(x) = A_0 + \sum_{r=1}^{\infty} (A_r \cos rx + B_r \sin rx)$$

$$= A_0 + A_1 \cos x + A_2 \cos 2x + \cdots + A_r \cos rx + \cdots B_1 \sin x + B_2 \sin 2x \cdots + B_r \sin rx + \cdots \quad (1)$$

This is known as a 'Fourier series'. The constants A_0, A_r and B_r are called Fourier coefficients.

Evaluation of A_0 – Multiplying equation(1) by dx and integrating from $-\pi$ to π, we get.

$$\int_{-\pi}^{\pi} f(x)dx = A_0 \int_{-\pi}^{\pi} dx + A_1 \int_{-\pi}^{\pi} \cos x\,dx + \cdots + A_r \int_{-\pi}^{\pi} \cos rx\,dx$$

$$+ \cdots B_1 \int_{-\pi}^{\pi} \sin x\,dx + \cdots + B_r \int_{-\pi}^{\pi} \sin rx\,dx$$

$$+ \cdots$$

$$= A_0(2\pi), \text{ all the other integrals being zero}$$

$$\therefore A_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\,dx \quad (2)$$

Evaluation of A_r :- Multiplying equation ① by $\cos rx\,dx$ and integrating from $-\pi$ to π we get

$$\int_{-\pi}^{\pi} f(x)\cos rx\,dx = A_0\int_{-\pi}^{\pi}\cos rx\,dx + A_1\int_{-\pi}^{\pi}\cos x\cos rx\,dx + \cdots$$

$$+ A_r\int_{-\pi}^{\pi}\cos^2 rx\,dx + \cdots - B_1\int_{-\pi}^{\pi}\sin x\cos rx\,dx$$

$$+ \cdots + B_r\int_{-\pi}^{\pi}\sin rx\cos rx\,dx + \cdots$$

$$= A_r\int_{-\pi}^{\pi}\cos^2 rx\,dx,$$ all the other integrals being zero

$$= A_r\int_{-\pi}^{\pi}\frac{1+\cos 2rx}{2}\,dx$$

$$= A_r\frac{1}{2}\left[x + \frac{\sin 2rx}{2r}\right]_{-\pi}^{\pi}$$

$$= A_r\frac{1}{2}\left[\pi + \frac{\sin 2r\pi}{2r} - (-\pi) - \frac{\sin(-2r\pi)}{2r}\right]$$

$$= A_r\frac{1}{2}\left[\pi + \pi\right] \qquad \because (\sin 2r\pi = 0)$$

$$= A_r(\pi)$$

$$\therefore A_r = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos rx\,dx \quad \text{——— ③}$$

Evaluation of B_r :- Multiplying equation ① by $\sin rx\,dx$ an integrating from $-\pi$ to π we get

$$B_r = \frac{1}{2}\int_{-\pi}^{\pi} f(x)\sin rx\,dx \quad \text{——— ④}$$

Because of the periodicity of the integrands the interval of integration in eqn ②, ③ and ④ may be replaced by any other interval of length 2π. Say, be the interval 0 to 2π.